Exercise 180

The number of hours of daylight in a northeast city is modeled by the function

$$N(t) = 12 + 3\sin\left[\frac{2\pi}{365}(t - 79)\right],$$

where t is the number of days after January 1.

a. Find the amplitude and period.

b. Determine the number of hours of daylight on the longest day of the year.

c. Determine the number of hours of daylight on the shortest day of the year.

d. Determine the number of hours of daylight 90 days after January 1.

e. Sketch the graph of the function for one period starting on January 1.

Solution

Part (a)

The amplitude is 3, the (positive) coefficient of the sine function. The period is

$$T = \frac{2\pi}{\frac{2\pi}{365}} = 365,$$

the number of days in a year.

Part (b)

The longest day of the year occurs when the sine is $+1$.

$$N(t) = 12 + 3(1) = 15 \text{ hours}$$

Part (c)

The shortest day of the year occurs when the sine is -1.

$$N(t) = 12 + 3(-1) = 9 \text{ hours}$$

Part (d)

Plug $t = 90$ into the formula to find the number of hours of daylight 90 days after January 1.

$$N(90) = 12 + 3\sin\left[\frac{2\pi}{365}(90 - 79)\right] \approx 12.6 \text{ hours},$$

Part (e)

Below is a graph of $N(t)$ versus t.